TEST #3 - CALCULATOR PORTION

Math 132 **Name:** ____________________

Problem	1	2	3	4	5	Total
Possible Score	25	25	30	40	30	150
Your Score						

SHOW ALL WORK. Any solution that is not accompanied by the appropriate work necessary for solving the problem will receive no credit. If you need more space, you may use the back of the page.

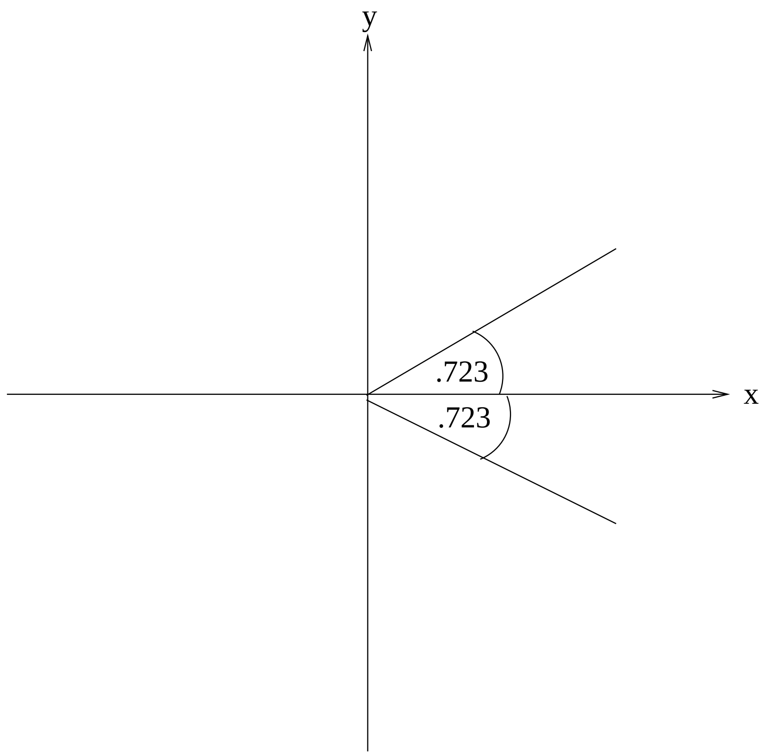
1. (25 pts) Solve the equation $4\cos(2x) + 1 = 4$ for $0 \le x \le \pi$.

SOLUTION: $4\cos(2x) + 1 = 4 \Rightarrow 4\cos(2x) = 3$

$$\Rightarrow \cos(2x) = .75$$

$$2x = \arccos(.75) \approx .723.$$

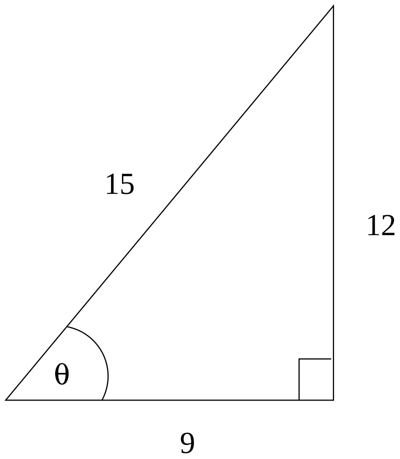
Since $0 \le x \le \pi$, we have $0 \le 2x \le 2\pi$ which gives the following figure.



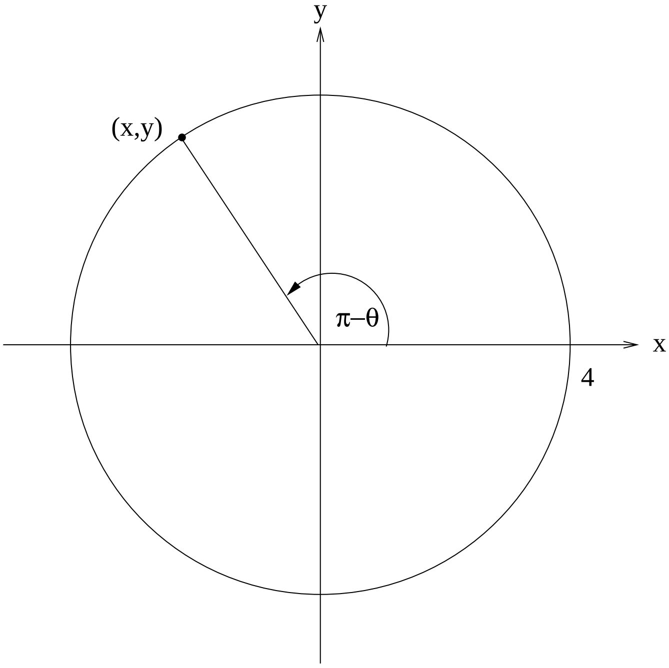
The angles $0 \le 2x \le 2\pi$ that correspond to the figure are $2x = .723, 2\pi - .723 \approx 5.56$.

Thus, $\boxed{x = .362, 2.78}$

2. (25 pts) Given the following right triangle containing the angle θ:



Find the coordinates of the point on the following circle of **radius 4** (**Hint:** think reference angles).



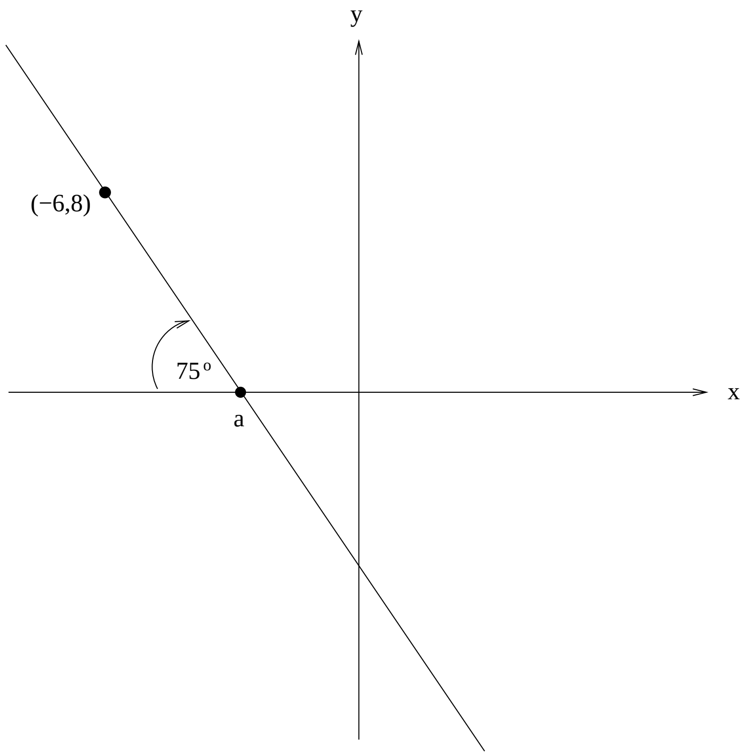
SOLUTION: Since the reference angle for $\pi - \theta$ is θ and since the point is on a circle of radius 4,

we have $\cos(\theta) = -\dfrac{x}{4} \Rightarrow x = -4\cos(\theta)$ and $\sin(\theta) = \dfrac{y}{4} \Rightarrow y = 4\sin(\theta)$.

From the right triangle, we see that $\cos(\theta) = \dfrac{9}{15} = .6$ and $\sin(\theta) = \dfrac{12}{15} = .8$.

Thus, $\boxed{x = -2.4 \text{ and } y = 3.2}$

3. (30 pts) Find the value of a in the below figure (**Round the answer to two decimal places**).



SOLUTION #1: Since a is on the negative part of the x-axis, we know $a < 0$.

We see from the above figure that $x + |a| = 6$, so $a = -(6 - x) = x - 6$.

$$\tan(75°) = \frac{8}{x} \Rightarrow x = \frac{8}{\tan(75°)}.$$

Thus, $a = \dfrac{8}{\tan(75°)} - 6 \approx \boxed{-3.86}$

OR

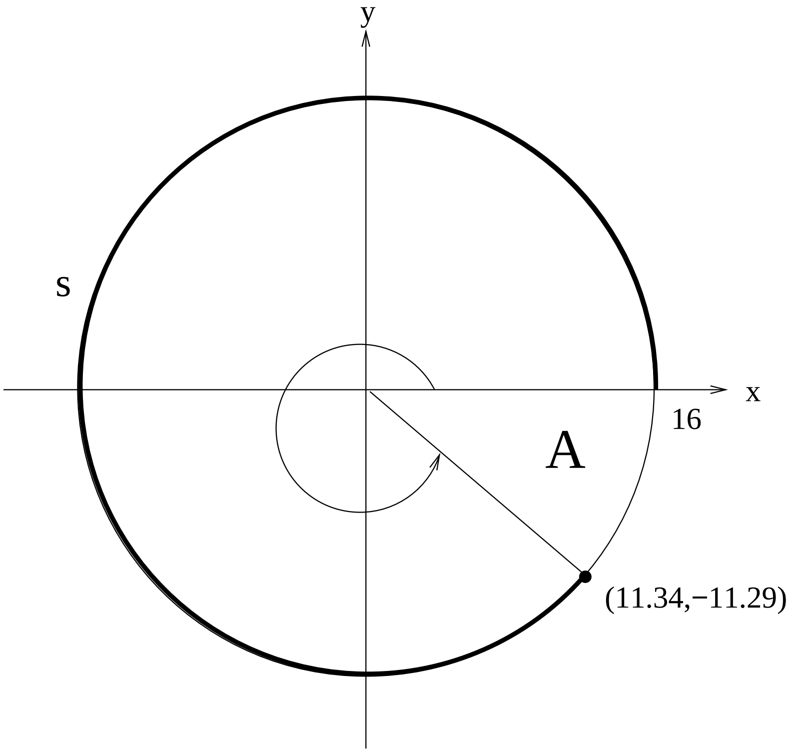
SOLUTION #2: Since a is the x-intercept of the line, we first find the equation of the line.

The slope is $m = \tan(105°)$ since $105°$ is the angle between the positive x-axis and the line.

Thus, the equation of the line is $y - 8 = \tan(105°)(x + 6)$, so letting $y = 0$, we get

$$-8 = \tan(105°)(x + 6) \Rightarrow x + 6 = -\frac{8}{\tan(105°)} \Rightarrow x = -6 - \frac{8}{\tan(105°)} \approx \boxed{-3.86}$$

4. In the figure below, the circle has radius $r = 16$.



(a) (20 pts) Find the length of the bold arc marked with an **s** in the above figure (**Round the answer to two decimal places**).

SOLUTION: The arc length is $s = r\theta$ where θ is the angle that determines the arc.

Since $r = 16$, we need only determine the value of θ.

We see from the circle that $\cos(\theta) = \dfrac{11.34}{16} \Rightarrow \theta = \arccos\left(\dfrac{11.34}{16}\right) \approx .783.$

However, this angle lies in the first quadrant, so $\theta = 2\pi - .783 \approx 5.5$.

Therefore, $s = 16(5.5) = \boxed{88}$

(b) (20 pts) Find the area of the sector marked with an **A** in the above figure (**Round the answer to two decimal places**).

SOLUTION: The area of the sector is $A = \dfrac{1}{2}\alpha r^2$ where α is the angle that determines the sector.

Since $\alpha + \theta = 2\pi$, we have $\alpha = 2\pi - \theta = 2\pi - 5.5 \approx .783$.

Thus, the area of the sector is $A = \dfrac{1}{2}(.783)(16)^2 \approx \boxed{100.22}$

TEST #3 - NO CALCULATOR PORTION

5. (30 pts) Sketch one period of the graph $y = -3\cos(5x + \pi) + 2$. Be sure to label the important x-values and y-values.