TEST #2 - CALCULATOR PORTION

Math 132 **Name:** ______________________________

Problem	1	2	3	4	5	Total
Possible Score	25	25	40	30	30	150
Your Score						

SHOW ALL WORK. Any solution that is not accompanied by the appropriate work necessary for solving the problem will receive no credit. If you need more space, you may use the back of the page.

1. (25 pts) Find the zeros of $f(x) = 9x^3 - 15x^2 + 2$. You must show all of your work at each step. You may not find the zeros by graphing the function with your graphing calculator and having the calculator compute the zeros for you.

   SOLUTION: By the Rational Zeros Theorem, we know the rational zeros of $f(x)$ will be of the form $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 3, \pm 9}$.

   By trial, we find $f\left(-\dfrac{1}{3}\right) = 0$.

   By using long division of polynomials, we have

$$
\begin{array}{r|rrrr}
 & & 3x^2 & -6x & +2 \\ \hline
3x+1 & 9x^3 & -15x^2 & & +2 \\
 & -9x^3 & +-3x^2 & & \\ \cline{2-3}
 & & -18x^2 & & \\
 & & +18x^2 & +6x & \\ \cline{3-4}
 & & & 6x & +2 \\
 & & & -6x & -2 \\ \cline{4-5}
 & & & & 0
\end{array}
$$

   Thus, $9x^3 - 15x^2 + 2 = (3x+1)(3x^2 - 6x + 2)$. We use the quadratic formula to find the zeros of $3x^2 - 6x + 2$.

   So, $x = \dfrac{6 \pm \sqrt{6^2 - (4)(3)(2)}}{2(3)} = \dfrac{6 \pm \sqrt{12}}{6} = 1 \pm \dfrac{\sqrt{3}}{3}$.

   Therefore, the zeros of $f(x)$ are $\boxed{-\dfrac{1}{3}, 1 + \dfrac{\sqrt{3}}{3}, 1 - \dfrac{\sqrt{3}}{3}}$

2. (25 pts) Find the x-values that satisfy $\dfrac{x}{x-4} \leq \dfrac{1}{x+5}$.

SOLUTION: $\dfrac{x}{x-4} \leq \dfrac{1}{x+5} \Rightarrow \dfrac{x}{x-4} - \dfrac{1}{x+5} \leq 0$

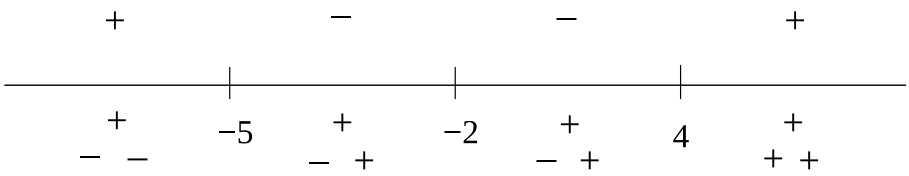
$$\Rightarrow \frac{x(x+5) - 1(x-4)}{(x-4)(x+5)} \leq 0$$

$$\Rightarrow \frac{x^2 + 5x - x + 4}{(x-4)(x+5)} \leq 0$$

$$\Rightarrow \frac{x^2 + 4x + 4}{(x-4)(x+5)} \leq 0$$

$$\Rightarrow \frac{(x+2)^2}{(x-4)(x+5)} \leq 0$$

The zeros of each factor are $x = -2$, $x = 4$, and $x = -5$, so we have the following:



Thus, $\boxed{(-5, 4)}$

3. Suppose L_1 is the line that passes through the points $(10, -3)$ and $(5, 5)$. Suppose L_2 is the line that passes through the points $(1, 2)$ and $(-7, 7)$.

(a) (16 pts) Are L_1 and L_2 parallel, perpendicular, or neither? **Show your work.**

SOLUTION: The slope for L_1 is $m_1 = \dfrac{-3-5}{10-5} = -\dfrac{8}{5}$.

The slope for L_2 is $m_2 = \dfrac{2-7}{1-(-7)} = -\dfrac{5}{8}$.

Since $m_1 \neq m_2$ and $m_1 \neq -\dfrac{1}{m_2}$, we have $\boxed{L_1 \text{ and } L_2 \text{ are neither}}$

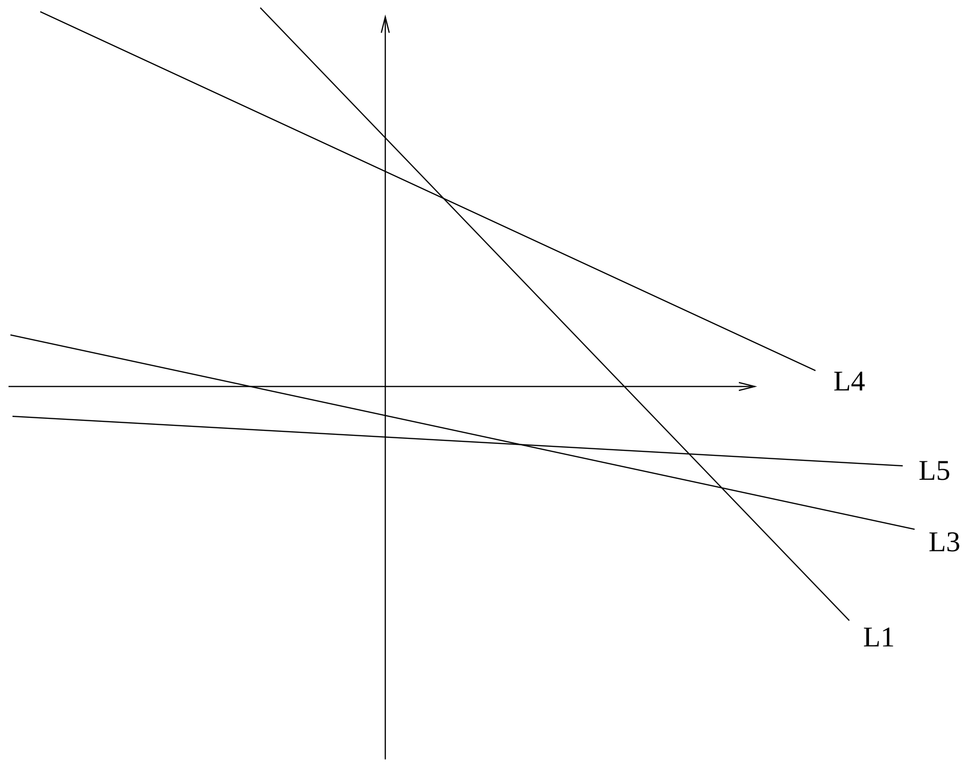
(b) (12 pts) Find the equation of the line L_3 that is parallel to L_2 and passes through the point $(1, -2)$.

SOLUTION: Since L_3 is parallel to L_2 which has slope $m_2 = -\dfrac{5}{8}$, the slope of L_3 is $m_3 = -\dfrac{5}{8}$.

Thus, the equation of L_3 is $\boxed{y + 2 = -\dfrac{5}{8}(x-1)}$ or $\boxed{y = -\dfrac{5}{8}x - \dfrac{11}{8}}$

PROBLEM #3 CONTINUED ON THE NEXT PAGE

(c) (12 pts) The following graph contains the lines L_1 and L_3 (which were described on the previous page) and lines L_4 and L_5 (which have not been described yet). Find the best equation for L_4 and L_5 from the list of equations below the graph (each line will have exactly one equation matched with it).



i. $y = -x - 3$

ii. $y = -x + 16$

iii. $y = -\frac{1}{4}x - 1$

iv. $y = -x + 10$ $\boxed{L_4}$

v. $y = -2x + 8$

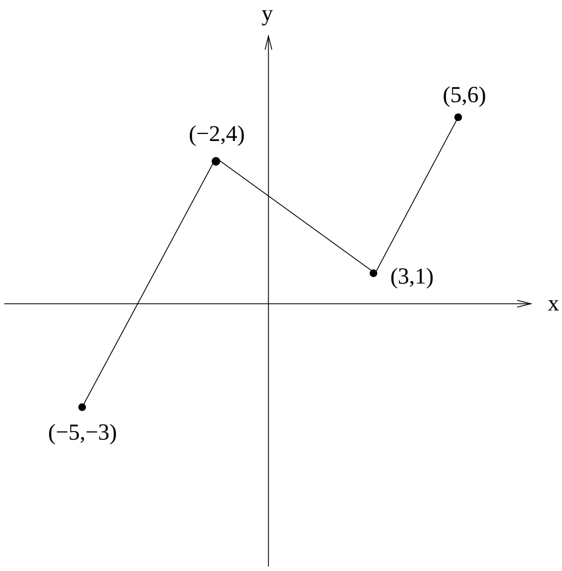
vi. $y = -\frac{1}{4}x - 3$ $\boxed{L_5}$

SOLUTION: The equation for L_1 is $y - 5 = -1.6(x - 5) \Rightarrow y = -1.6x + 13$, and the equation for L_3 is $y = -.625x - 1.375$.

So (ii) can't be L_4 since $16 > 13$, and (v) can't be L_4 since $-2 < -1.6$, so $\boxed{L_4 \text{ is (iv)}}$

(iii) can't be L_5 since $-1 < -1.375$, and (i) can't be L_5 since $-1 < -.625$, so $\boxed{L_5 \text{ is (vi)}}$

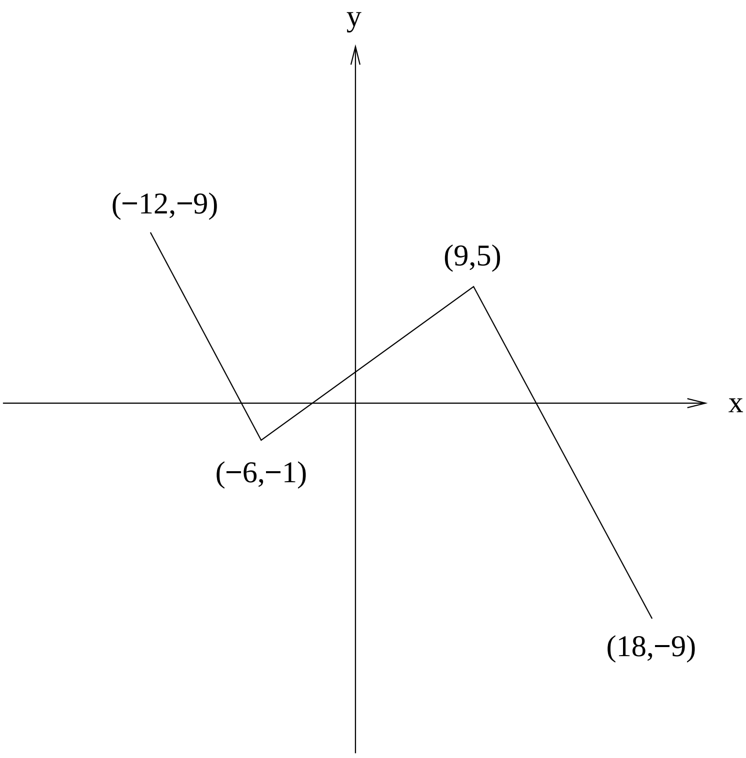
4. (30 pts) Below is the graph of $y = f(x)$. On the graph paper at the bottom of the page, graph the function $y = 2f\left(-\frac{1}{3}x + 1\right) - 3$.



SOLUTION: We transform each x-value by $-3(x-1)$, and each y-value by $2y-3$.

So, $(-5,-3) \Rightarrow (18,-9)$, $(-2,4) \Rightarrow (9,5)$, $(3,1) \Rightarrow (-6,-1)$, and $(5,6) \Rightarrow (-12,9)$.

The transformed graph is:



TEST #2 - NO CALCULATOR PORTION

Math 132 **Name:** ____________________

SHOW ALL WORK. Any solution that is not accompanied by the appropriate work necessary for solving the problem will receive no credit. If you need more space, you may use the back of the page.

5. (30 pts) Sketch the graph of $f(x) = (2x+5)^2(-6x^2+7x-2)$. Be sure to label all x-intercepts and y-intercepts.

SOLUTION: We factor to get $f(x) = (2x+5)^2(3x-2)(1-2x)$.

$f(0) = (5^2)(-2)(1) = -50$ is the y-intercept.

The x-intercepts are $x = -\dfrac{5}{2}, \dfrac{2}{3}, \dfrac{1}{2}$.

The graph bounces off the x-axis at $x = -\dfrac{5}{2}$ and crosses at $x = \dfrac{2}{3}, \dfrac{1}{2}$.

The long-run behavior is $(2x)^2(3x)(-2x) = -24x^4$ is that of an upsidedown parabola.

Thus, we have the following graph: