TEST #1

Math 132 **Name:** ______________________

Problem	1	2	3	4	5	Total
Possible Score	18	44	35	33	20	150
Your Score						

SHOW ALL WORK. Any solution that is not accompanied by the appropriate work necessary for solving the problem will receive no credit. If you need more space, you may use the back of the page.

1. (18 pts) Use long division of polynomials to divide $3x^4 - 5x^3 + 7x^2 + x - 12$ by $x^2 - 2x + 6$. Be sure to state the quotient (ie. the answer) and the remainder.

   SOLUTION: By using long division of polynomials, we have

$$
\begin{array}{r|rrrrr}
 & & & 3x^2 & +x & -9 \\
\hline
x^2 - 2x + 6 & 3x^4 & -5x^3 & +7x^2 & +x & -12 \\
 & -3x^4 & +6x^3 & -18x^2 & & \\
\hline
 & & x^3 & -11x^2 & +x & -12 \\
 & & -x^3 & +2x^2 & -6x & \\
\hline
 & & & -9x^2 & -5x & -12 \\
 & & & +9x^2 & -18x & +54 \\
\hline
 & & & & -23x & +42
\end{array}
$$

Thus, the quotient is $\boxed{3x^2 + x - 9}$ with a remainder of $\boxed{-23x + 42}$

2. A manufacturer of beds has a daily production cost of $C = 0.25x^2 - 10x + 800$ where C is the total cost in dollars and x is the number of beds produced.

(a) (20 pts) Complete the square to write C in the form $p(x+q)^2 + r$.

SOLUTION: $0.25x^2 - 10x + 800 = p(x+q)^2 + r = px^2 2pqx + (pq^2 + r)$ implies $p = 0.25$.

Also, $2pq = -10 \Rightarrow 2(.25)q = -10 \Rightarrow q = -20$.

Finally, $pq^2 + r = 800 \Rightarrow (.25)(-20)^2 + r = 800 \Rightarrow r = 700$.

Thus, $\boxed{C = 0.25(x-20)^2 + 700}$

(b) (12 pts) Use part (a) to determine how many beds must be produced per day to minimize the cost? What is this minimum cost?

SOLUTION: From part (a), we see that the vertex of the parabola is at $(20, 700)$, so the number of beds is $\boxed{x = 20}$ and the cost is $\boxed{C = 700}$

(c) (12 pts) Use part (a) to determine how many beds must be produced so that the cost of production is \$821?

SOLUTION: Letting $C = 821$ in part (a), we get

$$821 = 0.25(x-20)^2 + 700 \Rightarrow 121 = 0.25(x-20)^2$$

$$\Rightarrow 484 = (x-20)^2$$

$$\Rightarrow \pm 22 = x - 20$$

$$\Rightarrow -2 \text{ or } 42 = x$$

Since -2 beds is nonsense, $\boxed{x = 42}$

3. Suppose $\log_m(x) = 3.4$, $\log_m(y) = 5.8$, and $\log_m(z) = -2.3$.

(a) (15 pts) Calculate $\log_m\left(\frac{\sqrt{z}}{x^3y}\right)$ (**Round the answer to two decimal places**).

SOLUTION: $\log_m\left(\frac{\sqrt{z}}{x^3y}\right) = \log_m(\sqrt{z}) - \log_m(x^3y)$

$$= \log_m(z^{1/2}) - (\log_m(x^3) + \log_m(y))$$

$$= \frac{1}{2}\log_m(z) - 3\log_m(x) - \log_m(y)$$

$$= \frac{1}{2}(-2.3) - 3(3.4) - 5.8$$

$$= \boxed{-17.15}$$

(b) (20 pts) If $\ln(x) = 2.5$, then what is m (**Round the answer to two decimal places**)?

SOLUTION: From the logarithm rules, $\log_m(x) = \frac{\ln(x)}{\ln(m)}$.

Substituting $\log_m(x) = 3.4$ and $\ln(x) = 2.5$, we get $3.4 = \frac{2.5}{\ln(m)}$.

Therefore, $\ln(m) = \frac{2.5}{3.4} \Rightarrow \boxed{m = e^{2.5/3.4} \approx 2.09}$

4. Suppose $f(x) = \dfrac{3x-2}{1-5x}$ and $g(x) = \sqrt{4x+9}$.

(a) (15 pts) Evaluate $f \circ g(2)$.

SOLUTION:
$$\begin{aligned} f \circ g(2) &= f\big(g(2)\big) \\ &= f(\sqrt{17}) \\ &= \boxed{\frac{3\sqrt{17}-2}{1-5\sqrt{17}} \approx -.529} \end{aligned}$$

(b) (18 pts) Evaluate $g \circ f(x)$.

SOLUTION:
$$\begin{aligned} g \circ f(x) &= g\big(f(x)\big) \\ &= g\left(\frac{3x-2}{1-5x}\right) \\ &= \boxed{\sqrt{4\left(\frac{3x-2}{1-5x}\right)+9}} \end{aligned}$$

5. (20 pts) Solve $\log_2(2x-1)+\log_2(x-3)=3$.

SOLUTION: $\log_2(2x-1)+\log_2(x-3)=3 \Rightarrow \log_2\big((2x-1)(x-3)\big)=3$

$$\Rightarrow 2^{\log_2\big((2x-1)(x-3)\big)}=2^3$$

$$\Rightarrow (2x-1)(x-3)=8$$

$$\Rightarrow 2x^2-7x+3=8$$

$$\Rightarrow 2x^2-7x-5=0$$

By the quadratic formula, we get $x=\dfrac{7\pm\sqrt{(-7)^2-4(2)(-5)}}{2(2)}=\dfrac{7\pm\sqrt{89}}{4}$

However, $x=\dfrac{7-\sqrt{89}}{4}$ is an extraneous root, so $\boxed{x=\dfrac{7+\sqrt{89}}{4}}$