

Name _____

Dynamical Grapher - LAB #3

Iterated Function Systems: 2 Examples

Use the Dynamical Grapher applet to do the following tasks:

- 1) (**Generalized Sierpinski Triangle**) Recall that the Sierpinski Triangle is the invariant set for the IFS consisting of the 3 contractions with ratios $1/2$, $1/2$, and $1/2$ and centers $(0,0)$, $(\frac{1}{2}, \frac{\sqrt{3}}{2})$, and $(1,0)$. Using our formula

$$f(x, y) = (rx + (1 - r)a_1, ry + (1 - r)a_2),$$

(where $r > 0$ is the “ratio” for f and (a_1, a_2) is the “center” of f) we get the IFS:

$$f_1(x, y) = (.5x, .5y)$$

$$f_2(x, y) \approx (.5x + .25, .5y + .433)$$

$$f_3(x, y) = (.5x + .5, .5y)$$

- (a) If we use the same centers but use the ratio $1/b$ instead of $1/2$, the IFS produces a **generalized** Sierpinski Triangle. Write down the 3 functions in this IFS:

$$f_1(x, y) = \underline{\hspace{10em}}$$

$$f_2(x, y) = \underline{\hspace{10em}}$$

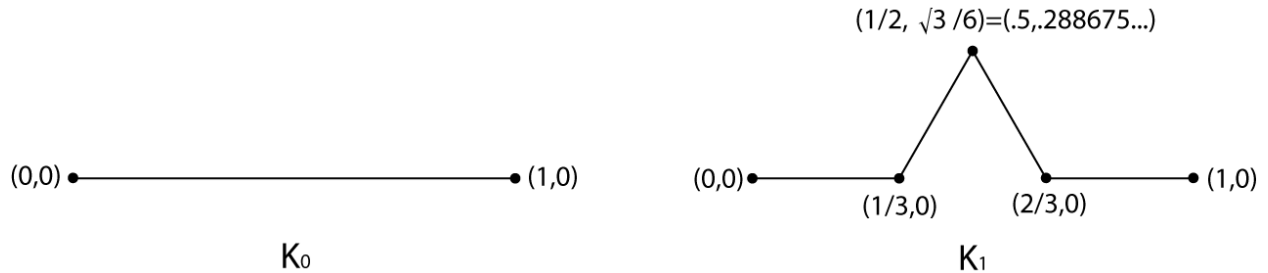
$$f_3(x, y) = \underline{\hspace{10em}}$$

(b) Enter the generalized IFS from (a) into the Dynamical Grapher. Set the parameter $b = 2$. Choose an initial curve (or point) and use the probabilistic function composition feature to graph the Sierpinski Triangle. (Go to the function tab marked "FC". Check "Probabilistic" and set the probabilities to $P(F1)=1$, $P(F2)=1$, and $P(F3)=1$. Set the number of iterations for "FC" on the curve C1 to range from 10 to 1000. (Make sure there is a C1 first!) Then hit "Calculate" followed by "GraphCurves.")

(c) Use the adjuster for "b" to increase "b". What happens to the fractal?

(d) As "b" goes to infinity, what do you think happens to the dimension of the fractal? Can you prove this mathematically?

2) (**Koch Curve**) Consider the fractal $K = \lim_{n \rightarrow \infty} K_n$, the initial stage of whose construction is illustrated as follows:



- (a) Find the 4 dilations making up the IFS having K as its invariant set. **Hint:** Find the contractions that map the line segment K_0 to each of the 4 line segments making up K_1 , i.e. contractions such that

$$f_1(K_0) \cup f_2(K_0) \cup f_3(K_0) \cup f_4(K_0) = K_1.$$

$$f_1(x, y) = \underline{\hspace{10cm}}$$

$$f_2(x, y) = \underline{\hspace{10cm}}$$

$$f_3(x, y) = \underline{\hspace{10cm}}$$

$$f_4(x, y) = \underline{\hspace{10cm}}$$

(b) Use the Dynamical Grapher to draw the line segment K_0 . Verify that the 4 functions map K_0 to the four smaller line segments making up K_1 .

(c) Choose an initial curve (or point) and use the probabilistic function composition feature to graph the fractal.

(d) Save your image (Under the "Actions" menu select "Take screen shot").